

# Homework 2

## General Instructions:

- Show your work for all questions. Even if you get the final answer wrong, you can get partial credit based on your approach to the problem.
- Submit electronically via Carmen. You can submit a scan of your work (PDF format) if you are doing this using pen and paper.

1. You are given a two-sided coin. You toss it a few times, and you get “HHH”. How would you go about estimating  $P(H)$  and  $P(T)$ .

(H = Heads, T = Tails)

- a. Using a Frequentist approach
  - b. Using a Bayesian approach
  - c. Which of (a) and (b) would you trust more in this case, and why.
2. You are given a **fair** two-sided coin (fair in the sense of Heads and Tails being equally likely, and each toss being independent). If you were to toss it two times, what would be the likelihood of each of the following:
    - a. HH
    - b. TT
    - c. At least one heads

3. For this question, use the joint probability table below. It involves three random variables:  $A \in \{a_1, a_2, a_3\}$ ,  $B \in \{b, \neg b\}$ , and  $C \in \{c, \neg c\}$

|       | $c$   |          | $\neg c$ |          |
|-------|-------|----------|----------|----------|
|       | $b$   | $\neg b$ | $b$      | $\neg b$ |
| $a_1$ | 0.012 | 0.09     | 0.049    | 0.063    |
| $a_2$ | 0.024 | 0.072    | 0.098    | 0.084    |
| $a_3$ | 0.084 | 0.018    | 0.343    | 0.063    |

- a. Reduce this to the joint probability tables between B and C only
  - b. Calculate  $P(\neg b | \neg c)$  and  $P(c)$ .
  - c. Is A conditionally independent of C, given B?
4. CVS starts offering Tuberculosis testing. You are given the following information:
    - The base likelihood of a random person having tuberculosis is 0.01%.
    - The tuberculosis test has an accuracy of 99%. i.e., if one has tuberculosis, the test result will be ‘positive’ 99% of the time; and if one doesn’t have tuberculosis, the test will be ‘negative’ 99% of the time.
    - a. If a test result is positive, what is the probability of the result being wrong?
    - b. For a random person, Tom, what is the probability of them not having Tuberculosis?
    - c. If Tom takes the Tuberculosis test and tests ‘positive’, what is the likelihood that Tom actually has tuberculosis?